FROM MANIPULATION TO WHEELED MOBILE MANIPULATION : ANALOGIES AND DIFFERENCES

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Abstract: a generic formulation of kinematics and instantaneous kinematics is proposed for wheeled mobile manipulators. It is compared with the classical kinematic modelling of robotic arms. In particular, we show that many tools of classical manipulation can be re-used in this new framework. Additionally, the specificities of this emerging type of manipulation are highlighted and the arising control problems are precised together with adequate definitions. So, this article provides comprehensive bases for the study of such systems.

Keywords: mobile manipulation, kinematics, instantaneous kinematics, nonholonomy

1. INTRODUCTION

In this paper, we consider mobile manipulators built from a wheeled mobile platform and a serial chain robotic arm. Thus, these systems combine manipulation and mobility capabilities. So the majority of the tasks they are (or will be) dedicated to require a certain level of coordination of the robotic arm and the platform. This is a specificity of mobile manipulation. In addition, in most tasks of manipulation, the user has to control the location of the tool or the grip of his robot – named as the end-effector and denoted by EE from now on. This might also be the case in mobile manipulation. From these two remarks, we find it very significant to model how the geometry and the motion of the *whole* mechanical structure act on the location and on the velocities of the EE.

Few contributions can be found in the literature on the problem kinematic and instantaneous kinematic modelling. Among them, we can cite some recent papers. Altafini (2001) deals with direct and inverse models of mobile manipulators equipped with omnidirectional platforms. However, from the kinematics and instantaneous kinematics point of view, these systems are very similar to the robotic arms and don't gen-

erate new manipulation concerns. Jakubia and Tchoń (2001) or Mohri et al. (2001) also address problems of kinematic modelling, but in the case of mobile manipulators equipped with a nonholonomic platform – i. e. subjected to constraints of rolling without slipping of the wheels on the ground. Both these studies consider the case of a particular system. More generally, modelling studies of mobile manipulators all apply, to our knowledge, to a system built from a particular type of platform, generally an unicycle, i. e. a platform with two independently driven wheels – Seraji (1993); Yamamoto and Yun (1994); Foulon et al. (1997); Mohri et al. (2001), seldom a car-like platform – Jakubia and Tchoń (2001). So, the analysis of mobile manipulators modelling suffer from this lack of genericity. It results a poor analysis of the specificities of mobile manipulation in comparison with classical manipulation. The purpose of this paper is to provide solutions to this problem.

To this end, manipulation and mobile robotics literature both provide tools to solve the modelling problem. On the one hand, the kinematics and instantaneous kinematics of robotic arms with a fixed base are now a very classical material. The associated notions of redundancy – Nakamura (1991), singularity

– Sciavicco and Siciliano (2000) – and manipulability –Yoshikawa (1990) – are also parts of the classical background of manipulation. On the other hand, wheeled mobile platforms were properly described and modelled by Campion et al. (1996). Though less classical and less used in the robotics community, these notions are of great interest in the case of wheeled mobile manipulators. In this paper we use the previous techniques to clarify how the well-known notions of classical manipulation adapt in the framework of mobile manipulation. This combination of the modellings inherited from manipulation and mobile robotics to model wheeled mobile manipulators, together with the comparison of classical and mobile manipulation are undoubtedly the major contributions of this article.

In section 2 we first recall the main notions attached to task description. In section 2.2, classical manipulation and the connection to kinematic modelling. Then, the modelling of wheeled mobile platforms is recalled in section 3. Kinematics and instantaneous kinematics modelling of wheeled mobile manipulators is developed in section 4. From this modelling, we discuss the issues of redundancy, singularity and manipulability. These properties are illustrated by comparing two systems equipped with the same robotic arm but with two different platforms.

2. TASKS AND MODELLING IN CLASSICAL MANIPULATION

2.1 Different types of tasks

A task is defined by the user in the so-called *operational space*. A point in this space is a *location*. It is characterized by a set of *operational coordinates* that correspond to the value of the position and the orientation of a frame attached to the EE at a particular point of this EE. Both those values are measured with respect to a fixed reference frame. The tasks are mainly of two types : *regulation* or *tracking*. In a task of regulation, the goal is to reach a desired value of the EE location. In a task of tracking, one needs to realize a given velocity of this location, i. e. a given operational velocity, to follow a prescribed operational motion. Note that the location of the EE can thus be defined in different ways, according to the task. For instance, for a planar problem, we will consider only the EE position and orientation in the plane.

2.2 Modelling of robotic arms

2.2.1. Description In classical manipulation, we consider a robotic arm built from n_a mobile bodies, supposed perfectly rigid and articulated by n_a rotoïd and/or prismatic joints. Let $\mathcal{R} = (O, \vec{x}, \vec{y}, \vec{z})$ be the reference frame with $\vec{z}$ vertical. The most usual way

to model a robotic arm consists in using the Denavit-Hartenberg modified parameters proposed by Khalil and Kleinfinger (1986). These parameters define the location of all the bodies of the robotic arm, i. e. its whole geometry. This parameterization associates a frame, denoted here by $\mathcal{R}_i = (O_i, \vec{x}_i, \vec{y}_i, \vec{z}_i)$, with $i = 0, 1, \dots, n_a$, to the i -th body of the robotic arm. So, the frame $\mathcal{R}_0$ is linked to the base. The center of the EE is denoted by O_{n_a+1} . Hence, both points O_{n_a} and O_{n_a+1} are linked to the EE.

The *robotic arm configuration* is known when the position of all its points in $\mathcal{R}_0$ are known (see for instance Neimark and Fufaev (1972)). It is defined by a vector $\mathbf{q}_a$ of n_a independent coordinates. These coordinates, called *generalized coordinates of the robotic arm*, characterize the values associated to the different joints: rotation angles for the rotoïd joints, translations for the prismatic ones. The configuration $\mathbf{q}_a = [q_{a1} \ q_{a2} \ \dots \ q_{an_a}]^T$ is an element of the *configuration space of the robotic arm*, denoted by $\mathcal{N}_a$.

In this article, we choose to define the *location of the EE* by a m_a -dimensional vector of **independent** coordinates, denoted by $\xi_a = [\xi_{a1} \ \xi_{a2} \ \dots \ \xi_{am_a}]^T$, which define the position and the orientation of the EE in $\mathcal{R}_0$. The set of all the locations constitutes the *operational space of the robotic arm EE*, denoted by $\mathcal{M}_a$. In the most general case we define : i) the position of the EE in $\mathcal{R}_0$ by the Cartesian coordinates of O_{n_a+1} : $[\xi_{a1} \ \xi_{a2} \ \xi_{a3}]^T$; ii) the orientation of the EE with the Euler classical angles – Paul (1981): $[\xi_{a4} \ \xi_{a5} \ \xi_{a6}]^T = [\psi \ \theta \ \varphi]^T$.

2.2.2. Kinematics The *kinematic model (KM) of the robotic arm* sets the location of its EE as a function of its configuration:

$$\begin{aligned} \mathbf{f}_a : \mathcal{N}_a &\longrightarrow \mathcal{M}_a \\ \mathbf{q}_a &\longmapsto \xi_a = \mathbf{f}_a(\mathbf{q}_a). \end{aligned}$$

2.2.3. Instantaneous kinematics The *instantaneous kinematic model (IKM) of the robotic arm* sets the derivative of its location – or operational velocity – as a function of the derivative of its configuration – or generalized velocity:

$$\begin{aligned} J_a(\mathbf{q}_a) : T_{\mathbf{q}_a}\mathcal{N}_a &\longrightarrow T_{\xi_a}\mathcal{M}_a \\ \dot{\mathbf{q}}_a &\longmapsto \dot{\xi}_a = J_a(\mathbf{q}_a)\dot{\mathbf{q}}_a, \end{aligned}$$

where $J_a(\mathbf{q}_a) = \frac{\partial \mathbf{f}_a}{\partial \mathbf{q}_a}(\mathbf{q}_a)$ is the $m_a \times n_a$ Jacobian matrix of $\mathbf{f}_a$. This linear map is the central object in kinematics.

2.2.4. Properties The rank of $J_a(\mathbf{q}_a)$ is denoted by $d_a(\mathbf{q}_a)$ for the configuration $\mathbf{q}_a$ and so we remark that $d_a(\mathbf{q}_a) \leq \min(m_a, n_a)$. Let D_a be the maximum value of $d_a(\mathbf{q}_a)$ over the workspace, and $R_a = n_a - D_a$ the minimum value of the dimension of the kernel of the previous map. We remark that $D_a \leq \min(m_a, n_a)$ and $R_a \geq 0$. Arm configurations

such that $d_a(\mathbf{q}_a) < D_a$ are called *singular configurations*, and robotics arms such that $R_a > 0$ are called *redundant*. In the sequel, the term *almost everywhere* (a. e.) means *everywhere except in the singular configurations*.

Now, we can point out some important properties of classical manipulation:

- **Task compatibility with the arm** As $D_a \leq \min(m_a, n_a)$ $D_a \leq m_a$. So, in order to choose **independent** operational coordinates to define the location it is necessary that $D_a = m_a$.
- **Regulation and redundancy** R_a is a. e. the dimension of the set of configurations that give a desired location.
- **Tracking**
 - **redundancy**: R_a is the dimension almost everywhere of the set of generalized velocities $\dot{\mathbf{q}}_a$ that give a desired operational velocity $\dot{\xi}_a$. R is also the dimension of the *self motion manifold* (see Murray et al. (1994)) i. e. the manifold whose tangent vectors are in the kernel of J_a ;
 - **singularities**: they lower the dimension of the range space of J_a thus turning impossible the tracking of some operational velocities;
 - **manipulability**: singular values of J_a and associated eigenvectors allow to give a measure of anisotropy of the operational velocities when generalized velocities are bounded inside a unit ball, for a given configuration.

Additionally, we can remark that

- the dimension of the set of solutions that give a desired location or a desired operational velocity is a. e. of the same dimension R_a . Thus, redundancy means that the set of generalized solutions is of dimension R_a both in tracking and regulation tasks;
- the system is *holonomic*, i. e. that the generalized velocities are independent. As a consequence it will be useful to define the *vector of the kinematic control of the robotic arm* by:

$$\mathbf{u}_a = \dot{\mathbf{q}}_a. \quad (1)$$

3. WHEELED MOBILE PLATFORMS

The wheeled mobile platforms are described and modelled by Campion et al. (1996). Only minor changes have been done in the sequel to be consistent with the modelling of mobile manipulators.

3.1 Description

Let $\mathcal{R}' = (O', \vec{x}', \vec{y}', \vec{z}')$ be a mobile frame linked to the platform (see Fig. 2). The origin of $\mathcal{R}'$ is

usually chosen as a remarkable point of this platform (e.g. the midpoint of the rear axle). The *location*¹ of the platform is given by a vector ξ_p of $m_p = 3$ operational coordinates, which define its position and orientation in $\mathcal{R}$. We write $\xi_p = [x \ y \ \vartheta]^T$, where x and y are respectively the abscissa and the ordinate of O' in $\mathcal{R}$ and ϑ the angle $(\vec{x}, \vec{x}')$. The set of all the locations constitutes the *operational space of the platform*, denoted by $\mathcal{M}_p$.

The platforms wheels can be classified in four types (see Fig. 1):

- the *fixed wheels* for which the axle has a fixed direction;
- the *steering wheels*, for which the orientation axis passes through the center of the wheel;
- the *castor wheels*, for which the orientation axis does not pass through the center of the wheel;
- the *Swedish wheels*, which are similar to the fixed wheels, with the exception of the additional parameter γ that describes the direction, with respect to the wheels plane, of the zero component of the velocity at the contact point.

Fig. 1. Different types of wheels

It is assumed that the wheels always keep their shape – which is sensible in indoor robotics – and that there is always *rolling without slipping* (r.w.s.) of the wheels on the ground.

Let $N = N_f + N_s + N_c + N_{sw}$ be the total number of wheels. N_f , N_s , N_c , N_{sw} represent respectively the number of fixed, steering, castor and Swedish wheels. From now on, these indexes correspond to the different types of wheels. Let φ_f , φ_s , φ_c , φ_{sw} be the vectors giving the rotation angles of the wheels about

¹ This quantity is not distinguished from the *posture* in Campion et al. (1996). Nonetheless, it seems important to us to isolate the part of the posture that is related to the EE location, on one side, and that will appear as the range of a linear map defined over the control of mobility (see hereafter).

their horizontal axle: all of them are variables. Let β_f , β_s , β_c and β_{sw} be the vectors giving the orientation of the wheels: β_f and β_{sw} are constants whereas β_s and β_c are variables. The N -dimensional rotation vector is $\varphi = [\varphi_f^T \varphi_s^T \varphi_c^T \varphi_{sw}^T]^T$. The *platform configuration* is known when the position of all its points in $\mathcal{R}$ are known. It is given by the n_p -dimensional vector $\mathbf{q}_p = [\varphi^T \xi_p^T \beta_s^T \beta_c^T]^T$, with $n_p = N + m_p + N_s + N_c$. The set of all the configurations constitutes the *configuration space of the platform*, denoted by $\mathcal{N}_p$.

The platforms are then described by the type of the different wheels, and by their geometry: the angles α , β , γ and the lengths l , l' and r (see Fig. 1).

3.2 Instantaneous kinematics

Let $R(\vartheta)$ be the 3-order rotation matrix expressing the orientation of $\mathcal{R}'$ with respect to $\mathcal{R}$. According to Fig. 1 the r.w.s. constraints can always be written:

$$J_1(\beta_s, \beta_c)R^T(\vartheta)\dot{\xi}_p + J_2\dot{\varphi} = \mathbf{0}, \quad (2)$$

$$C_1(\beta_s, \beta_c)R^T(\vartheta)\dot{\xi}_p + C_2\dot{\beta}_c = \mathbf{0}, \quad (3)$$

where the associated matrices are given in Campion et al. (1996). Equation (2) expresses the constraint in the vertical plane of the wheel and equation (3) in the vertical plane orthogonal to the wheel; they are said to be *nonholonomic* since they cannot be integrated.

If we take into account the structures of C_1 and C_2 we can separate (3) in two parts:

$$C_1^*(\beta_s)R^T(\vartheta)\dot{\xi}_p = \mathbf{0}, \quad (4)$$

$$C_{1c}(\beta_c)R^T(\vartheta)\dot{\xi}_p + C_{2c}\dot{\beta}_c = \mathbf{0}. \quad (5)$$

Equation (4) is such that $R^T(\vartheta)\dot{\xi}_p$ must be an element of $C_1^*(\beta_s)$ null space, denoted by $\text{Ker}(C_1^*(\beta_s))$. Let $\Sigma(\beta_s)$ be a matrix the columns of which are a basis of this null space. It is $3 \times \delta_{m_p}$ -dimensional, with: $\delta_{m_p} = \dim(\text{Ker}(C_1^*(\beta_s))) = 3 - \text{rank } C_1^*(\beta_s)$; δ_{m_p} is termed as the *degree of mobility of the platform*. From the previous calculus, we conclude that there exists a δ_{m_p} -dimensional vector η_p such that:

$$\dot{\xi}_p = R(\vartheta)\Sigma(\beta_s)\eta_p. \quad (6)$$

We will term it as the *control of mobility of the platform* and equation (6) forms the *instantaneous location kinematic model (ILKM) of the platform*. It sets the derivative of the platform location as a function of the control of mobility, for a given configuration.

When there are steering wheels, instantaneous kinematics are not completely described by the (ILKM) in the sense that this model does not fix the whole generalized velocity. This feature is revealed by the structure of equation (4) that can be decomposed in two parts :

$$C_{1f}R^T(\vartheta)\dot{\xi}_p = \mathbf{0}, \quad C_{1s}(\beta_s)R^T(\vartheta)\dot{\xi}_p = \mathbf{0}.$$

Then, $\delta_{s_p} = \text{rank } C_{1s}(\beta_s)$ is the *degree of steerability* and $\delta_{M_p} = \delta_{m_p} + \delta_{s_p} = \delta_{m_p} + N_s$ the *degree of manoeuvrability of the platform*.

We assume that we control the velocity of the steering wheels around their orientation axis. Thus, it is necessary to have $N_s = \delta_{s_p}$ to control independently the N_s steering wheels. As a consequence, the δ_{s_p} -dimensional vector ζ_p is termed as the *control of steerability of the platform*:

$$\dot{\beta}_s = \zeta_p, \quad (7)$$

With Campion et al. (1996), let the $3 + N_s$ -dimensional vector $z_p = [\xi_p^T \beta_s^T]^T$ be the *posture of the platform*². If we define the *control of manoeuvrability of the platform* by the δ_{M_p} -dimensional vector $u_p = [\eta_p^T \zeta_p^T]^T$, we get:

$$\dot{z}_p = B_p(\vartheta, \beta_s)u_p,$$

with:

$$B_p(\vartheta, \beta_s) = \begin{bmatrix} R(\vartheta)\Sigma(\beta_s) & 0 \\ 0 & I_{N_s} \end{bmatrix},$$

where I_{N_s} is the N_s -order identity matrix. This is the *instantaneous posture kinematic model (IPKM)*. It can be shown that this model is irreducible (see de Wit et al. (1996)). It constitutes the minimal model that allows to obtain the whole generalized velocity in the form:

$$\dot{\mathbf{q}}_p = S_p(\vartheta, \beta_s, \beta_c)u_p, \quad (8)$$

by taking into account equations (6) and (7) in (2) and (3). This equation relates the derivative of the platform configuration, for a given configuration, to its control of manoeuvrability. It is termed as the *instantaneous configuration kinematic model (ICKM) of the platform*.

From now on, we assume that the platform always has a number of actuators equal to its degree of manoeuvrability.

4. MODELLING OF MOBILE MANIPULATORS

4.1 Description

We consider the general case of a mobile manipulator built from a platform equipped with an on-board robotic arm, introduced respectively in paragraph 3 and 2.2. The Cartesian coordinates of O_0 in $\mathcal{R}'$ are given by $[a \ b \ 0]^T$ (see Fig. 2). Remark that this choice is always possible whatever the position and orientation of the base of the robotic arm on the platform.

The *mobile manipulator configuration* is known when the position of all its points in $\mathcal{R}$ are known. It is

² Posture and location need to be distinguished only when the platform has steering wheels. When there is no steering wheels, posture and location are the same notion.

Fig. 2. Geometry of mobile manipulators

defined by a vector $\mathbf{q} = [q_1 \ q_2 \ \dots \ q_n]^T = [\mathbf{q}_a^T \ \mathbf{q}_p^T]^T$ of n independent coordinates, called *generalized coordinates of the mobile manipulator*. We notice that $n = n_a + n_p$, where n_a and n_p are respectively the dimensions of the generalized spaces associated to the robotic arm and to the platform. The configuration $\mathbf{q}$ is an element of the *configuration space of the mobile manipulator*, denoted by $\mathcal{N}$.

The *location of the mobile manipulator* EE is given by the m -dimensional vector $\boldsymbol{\xi} = [\xi_1 \ \xi_2 \ \dots \ \xi_m]^T$. Its m coordinates are the *operational coordinates of the mobile manipulator*. They define the position and the orientation of the EE in $\mathcal{R}$. The set of all the locations constitutes the *operational space of the mobile manipulator*, denoted by $\mathcal{M}$. In the most general case we define: i) the position of the EE in $\mathcal{R}$ by the Cartesian coordinates of O_{n_a+1} : $[\xi_1 \ \xi_2 \ \xi_3]^T$; ii) the orientation of the EE with the Euler classical angles: $[\xi_4 \ \xi_5 \ \xi_6]^T = [\beta \ \theta \ \phi]^T$. Remark that $\xi_4 = \vartheta + \psi$, $\xi_5 = \xi_{a5}$ and $\xi_6 = \xi_{a6}$. These properties justify the choice of the Euler classical angles.

4.1.1. Kinematics Concerning kinematics we first notice that to a given EE location can be associated any value of φ and β_s i.e. an infinity of configurations. Thus, we will consider only a subset of the mobile manipulator configuration: the *mobile manipulator partial configuration* which is defined by the $n_a + m_p$ -dimensional vector $\mathbf{p} = [p_1 \ p_2 \ \dots \ p_{n_a+m_p}]^T = [\mathbf{q}_a^T \ \boldsymbol{\xi}_p^T]^T$, that affects the EE location. The components of this vector are called *partial generalized coordinates of the mobile manipulator*. The (geometric) redundancy will be defined from this modelling.

The *kinematic model (KM) of the mobile manipulator* sets the location of its EE as a function of the robotic arm configuration and of the platform location:

$$\begin{aligned} \mathbf{f}: \quad \mathcal{N}_a \times \mathcal{M}_p &\longrightarrow \mathcal{M} \\ \mathbf{p} = (\mathbf{q}_a, \boldsymbol{\xi}_p) &\longmapsto \boldsymbol{\xi} = \mathbf{f}(\mathbf{q}_a, \boldsymbol{\xi}_p) = \mathbf{f}(\mathbf{p}). \end{aligned}$$

The shape of the mobile manipulators we are studying is shown in Fig. 2, regardless of the type of wheels. For any wheeled platform with location $\boldsymbol{\xi}_p = [x \ y \ \vartheta]^T$,

equipped with a robotic arm with configuration $\mathbf{q}_a$ and location $\boldsymbol{\xi}_a$, the position of the mobile manipulator EE in $\mathcal{R}$ is:

$$\begin{aligned} \xi_1 &= x + (a + \xi_{a1})C_\vartheta - (b + \xi_{a2})S_\vartheta, \\ \xi_2 &= y + (a + \xi_{a1})S_\vartheta + (b + \xi_{a2})C_\vartheta, \\ \xi_3 &= \xi_{a3} \end{aligned} \quad (9)$$

and its orientation is:

$$\begin{aligned} \xi_4 &= \beta = \vartheta + \psi, \\ \xi_5 &= \theta, \\ \xi_6 &= \phi. \end{aligned} \quad (10)$$

From this model, we are led to reconsider the different notions developed in robotic arms modelling for the wheeled mobile manipulators. First, it can be remarked that the different generalized coordinates of a mobile manipulator play a different role. Indeed:

- Generally, we are not interested in imposing the angular values φ of the wheels but rather in fixing the location – or possibly the posture – of the platform;
- When the system has steering wheels the EE location is independent of the orientation value of the steering wheels; the location of the platform is the only component of its geometry that determines the EE location,

In the two next paragraphs, the role of the control of mobility – the only one that acts on the instantaneous velocities of the EE – and of the control of manoeuvrability will be emphasized and connected to the different models and related notions.

4.1.2. Instantaneous kinematics model The *instantaneous kinematic model (IKM) of the mobile manipulator* sets the derivative of its location as a function of the derivatives of the robotic arm configuration and of the location of the platform:

$$\dot{\boldsymbol{\xi}} = \frac{\partial \mathbf{f}}{\partial \mathbf{q}_a}(\mathbf{q}_a, \vartheta) \dot{\mathbf{q}}_a + \frac{\partial \mathbf{f}}{\partial \boldsymbol{\xi}_p}(\mathbf{q}_a, \vartheta) \dot{\boldsymbol{\xi}}_p = \mathbf{J}(\mathbf{q}_a, \vartheta) \dot{\mathbf{p}}, \quad (11)$$

where $\mathbf{J}(\mathbf{q}_a, \vartheta) = \frac{\partial \mathbf{f}}{\partial \mathbf{p}}(\mathbf{q}_a, \vartheta)$ is the $m \times (n_a + m_p)$ Jacobian matrix of $\mathbf{f}$. We can notice that $\mathbf{f}$ is a function of $\mathbf{q}_a$ and $\boldsymbol{\xi}_p$, whereas its partial derivatives only depend on $\mathbf{q}_a$ and ϑ . Actually, from equations (9) and (10):

$$\frac{\partial \mathbf{f}}{\partial \mathbf{q}_a}(\mathbf{q}_a, \vartheta) = \begin{bmatrix} J_{a1,*}C_\vartheta - J_{a2,*}S_\vartheta \\ J_{a1,*}S_\vartheta + J_{a2,*}C_\vartheta \\ J_{a3,*} \\ J_{a4,*} \\ J_{a5,*} \\ J_{a6,*} \end{bmatrix},$$

$J_{a_{k,*}}$ being the k -th line of the robotic arm Jacobian matrix and:

$$\frac{\partial \mathbf{f}}{\partial \boldsymbol{\xi}_p}(\mathbf{q}_a, \vartheta) = \begin{bmatrix} 1 & 0 & -((a + \xi_{a1})S_\vartheta + (b + \xi_{a2})C_\vartheta) \\ 0 & 1 & (a + \xi_{a1})C_\vartheta - (b + \xi_{a2})S_\vartheta \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Let $d(\mathbf{q}_a, \vartheta)$ be the rank of $J(\mathbf{q}_a, \vartheta)$ for the partial configuration $\mathbf{p}$. Remark that $d(\mathbf{q}_a, \vartheta) \leq \min(m, n_a + m_p)$. Let D be the maximum value of $d(\mathbf{q}_a, \vartheta)$, and $R = n_a + m_p - D$ the minimum value of the dimension of the kernel of the previous map (remark that $D \leq \min(m, n_a + m_p)$ and $R \geq 0$). As in the robotic arm case :

- $d(\mathbf{q}_a, \vartheta) < D$ corresponds to singular partial configurations;
- R is the dimension *a. e.* of the set of partial configurations $\mathbf{p}$ (and not configurations $\mathbf{q}!$) that give a desired EE location. It is then the degree of *geometric redundancy*;

4.1.3. Instantaneous location, posture and configuration kinematic model In fact the map (11) does not model the *admissible* velocities of the EE since it does not consider the nonholonomic constraint of the platform. This is done by considering the *control of mobility of the mobile manipulator*: $\boldsymbol{\eta} = [\mathbf{u}_a^T \ \boldsymbol{\eta}_p^T]^T$. Its dimension $\delta_m = n_a + \delta_{m_p}$ is termed as the *degree of mobility of the mobile manipulator*. The *instantaneous location kinematic model* (ILKM) of the mobile manipulator sets the derivative of its location as a function of its control of mobility. According to (1) and (6), equation (11) writes:

$$\dot{\mathbf{x}} = \frac{\partial \mathbf{f}}{\partial \mathbf{q}_a}(\mathbf{q}_a, \vartheta) \mathbf{u}_a + \frac{\partial \mathbf{f}}{\partial \boldsymbol{\xi}_p}(\mathbf{q}_a, \vartheta) R(\vartheta) \Sigma(\boldsymbol{\beta}_s) \boldsymbol{\eta}_p.$$

So the ILKM writes:

$$\dot{\mathbf{x}} = \bar{J}(\mathbf{q}_a, \vartheta, \boldsymbol{\beta}_s) \boldsymbol{\eta}, \quad (12)$$

with $\bar{J}(\mathbf{q}_a, \vartheta, \boldsymbol{\beta}_s) = [\frac{\partial \mathbf{f}}{\partial \mathbf{q}_a}(\mathbf{q}_a, \vartheta) \ \frac{\partial \mathbf{f}}{\partial \boldsymbol{\xi}_p}(\mathbf{q}_a, \vartheta) R(\vartheta) \Sigma(\boldsymbol{\beta}_s)]$ a $m \times \delta_m$ matrix, which is **not** Jacobian.

From this model, we again have to reconsider the different notions developed in robotic arms modelling for the wheeled mobile manipulators:

- the ability to produce EE velocities does not depend on φ . Considering that φ doesn't influence either mobile manipulators kinematics, we can conclude that two mobile manipulators for which only the angles φ differ share the same properties and are not distinguishable from the task viewpoint;
- when the system has steering wheels the ability to produce EE velocities depends on the orientation value $\boldsymbol{\beta}_s$ but not on its derivative. In other words it does not depend on the control of steerability ζ_p of the platform. Moreover, when no dynamic phenomena are taken into account, the steerability control is independent of any generalized coordinate.

To take into account the control of steerability, we use (12) and (7), to set the *instantaneous posture kinematic model* (IPKM) of a mobile manipulator:

$$\begin{aligned} \dot{\boldsymbol{\xi}} &= \bar{J}(\mathbf{q}_a, \vartheta, \boldsymbol{\beta}_s) \boldsymbol{\eta} \\ \dot{\boldsymbol{\beta}}_s &= \zeta_p \end{aligned}$$

Let the $(m + N_s)$ – dimensional vector $\mathbf{z} = [\boldsymbol{\xi}^T \ \boldsymbol{\beta}_s^T]^T$ be the *posture of the mobile manipulator*. If we define the *control of manoeuvrability of the mobile manipulator* by the δ_M –dimensional vector $\mathbf{u} = [\mathbf{u}_a^T \ \mathbf{u}_p^T]^T = [\boldsymbol{\eta}^T \ \zeta_p^T]^T$ such that $\delta_M = n_a + \delta_{m_p} = \delta_m + \delta_{s_p}$ (δ_M is termed as the *degree of manoeuvrability of the mobile manipulator*) we get:

$$\dot{\mathbf{z}} = B(\mathbf{q}_a, \vartheta, \boldsymbol{\beta}_s) \mathbf{u},$$

with: $B(\mathbf{q}_a, \vartheta, \boldsymbol{\beta}_s) = \begin{bmatrix} \bar{J}(\mathbf{q}_a, \vartheta, \boldsymbol{\beta}_s) & 0 \\ 0 & I_{N_s} \end{bmatrix}$. It can be shown that this model is irreducible. It constitutes the minimal model that allows to obtain the whole generalized velocity that can be written in the form:

$$\dot{\mathbf{q}} = S(\mathbf{q}) \mathbf{u}. \quad (13)$$

This equation relates the derivative of the mobile manipulator configuration, for a given configuration, to its control of manoeuvrability. It is termed as the *instantaneous configuration kinematic model* (ICKM) of the mobile manipulator.

4.1.4. Properties Let $\bar{d}$ be the rank of $\bar{J}(\mathbf{q}_a, \vartheta, \boldsymbol{\beta}_s)$ and $\bar{D}$ its maximum value. When there is no steering wheel ($\delta_{s_p} = 0$), the map $\bar{J}(\mathbf{q}_a, \vartheta, \boldsymbol{\beta}_s)$ can be used in lieu of the classical Jacobian:

- $\bar{R} = \delta_m - \bar{D}$ is the dimension of the set of solutions $\boldsymbol{\eta}$ that realize the same $\dot{\mathbf{x}}$ for the given configuration. Thus, there is a *velocity redundancy* with degree $\bar{R}$ and $\bar{R}$ determines the nature of the set of generalized velocity that are solution : *finite or infinite dimensional*.
- Manipulability can be defined by considering how $\bar{J}(\mathbf{q}_a, \vartheta, \boldsymbol{\beta}_s)$ maps the unit ball $\|\boldsymbol{\eta}\| \leq 1$ in the space of operational velocities,
- In the same way, singularities of $\bar{J}(\mathbf{q}_a, \vartheta, \boldsymbol{\beta}_s)$ have to be known in order to check *consistency* or to use algorithms that do not assume full rank of the linear map.

Nonetheless, due to nonholonomic constraints, the case $\bar{D} < D$ is possible (consider a mobile manipulator with $n_a = 0 \dots$ but there are real cases not so trivial!). In that case, some EE velocities $\dot{\mathbf{x}}$ (where $\boldsymbol{\xi}$ is a set of independent parameters from the geometric viewpoint i. e. $\boldsymbol{\xi}$ is of dimension D) are not realizable by the mobile manipulator.

Thus, it is possible to have for the same mobile manipulator the following properties (impossible in classical manipulation): all the EE locations realizable in a operational space of dim D (accessibility), possibly with an infinity of solutions ($n_a + m_p > D$) (geometric redundancy), some velocities not realizable along this space ($\bar{D} < D$) (no consistency), and infinity of solutions $\boldsymbol{\eta}$ for a realizable $\dot{\mathbf{x}}$ ($\bar{R} \neq 0$) (velocity

redundancy). Such an example is given in Bayle et al. (To appear).

When there are steering wheels ($\delta_{s_p} \neq 0$), all these properties between the mobility control vector and the EE velocity hold but redundancy deserves a more detailed treatment. In fact, when $\delta_{s_p} \neq 0$, $\bar{R} = 0$ does not mean that there is one and only one generalized velocity that corresponds to the desired EE velocity since the steering velocity ζ_p can be chosen independently. Along a trajectory characterized by an initial configuration q^0 (and its associated ξ^0) and $\dot{\xi}(t)$, there is an infinity of $q(t)$, even if $\bar{R} = 0$, depending on the law chosen for ζ_p . Thus, velocity redundancy concerns only mobility control vector and determining the steering law cannot be cast in the same mould as the classic kinematic control scheme for robotic arms.

4.2 Example

Fig. 3. Planar mobile manipulator with a car-like platform

We consider the mobile manipulator shown in Fig. 3 and we compare it to the mobile manipulator made with the same robotic arm and a platform without steering wheels (in both cases $b = 0$ for the sake of simplicity).

The EE location in the plane is defined as follows. The position of the point O_3 on the EE with respect to the frame $\mathcal{R}$ (resp. $\mathcal{R}_0$) is given by its Cartesian coordinates ξ_1 and ξ_2 (resp. ξ_{a1} and ξ_{a2}) and the orientation is given by the angle $\xi_3 = \beta$ (resp. $\xi_{a3} = \psi$).

The robotic arm KM is:

$$\begin{aligned}\xi_{a1} &= a_1 C_1 + a_2 C_{12}, \\ \xi_{a2} &= a_1 S_1 + a_2 S_{12}, \\ \xi_{a3} &= q_{a1} + q_{a2}\end{aligned}\quad (14)$$

where: $C_i = \cos q_{ai}$, $S_i = \sin q_{ai}$, $C_{ij} = \cos(q_{ai} + q_{aj})$ and $S_{ij} = \sin(q_{ai} + q_{aj})$, $\forall i, j = 1, 2$.

Thus, according to (9), both mobile manipulators share: • the same KM:

$$\begin{aligned}\xi_1 &= x + (a + a_1 C_1 + a_2 C_{12})C_\vartheta - (a_1 S_1 + a_2 S_{12})S_\vartheta, \\ \xi_2 &= y + (a + a_1 C_1 + a_2 C_{12})S_\vartheta + (a_1 S_1 + a_2 S_{12})C_\vartheta, \\ \xi_3 &= \vartheta + q_{a1} + q_{a2}\end{aligned}\quad (15)$$

• the first two columns of $\bar{J}$:

$$\frac{\partial f}{\partial q_a} = [\bar{J}_1 \mid \bar{J}_2] = \begin{bmatrix} D_2 & D_1 \\ D_4 & D_3 \\ 1 & 1 \end{bmatrix}$$

where

$$\begin{aligned}D_1 &= -a_2 S_{12\vartheta} \\ D_2 &= D_1 - a_1 S_{1\vartheta} \\ D_3 &= a_2 C_{12\vartheta} \\ D_4 &= D_3 + a_1 C_{1\vartheta},\end{aligned}$$

• and the matrix $\frac{\partial f}{\partial \xi_p}(q_a, \vartheta)$:

$$\frac{\partial f}{\partial \xi_p}(q_a, \vartheta) = \begin{bmatrix} 1 & 0 & D_5 \\ 0 & 1 & D_6 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where $D_5 = D_2 - a S_\vartheta$ and $D_6 = D_4 + a C_\vartheta$.

Thus, the construction of $\bar{J}(q_a, \vartheta, \beta_s)$ differs only when we write ξ_p as a function of the control of mobility η_p .

	car	unicycle
$R(\vartheta)\Sigma$	$\begin{bmatrix} D_7 C_\vartheta \\ D_7 S_\vartheta \\ C_{\beta_3} \end{bmatrix}$	$\begin{bmatrix} C_\vartheta & 0 \\ S_\vartheta & 0 \\ 0 & 1 \end{bmatrix}$

where $D_7 = -D S_{\beta_3}$.

Finally, we get:

• for the car-like mounted mobile manipulator:

$$\begin{aligned}\dot{\xi} &= \bar{J}(q_a, \vartheta, \beta_s)\eta \\ \dot{\beta}_3 &= \zeta.\end{aligned}$$

with: $\bar{J}(q_{a1}, q_{a2}, \vartheta, \beta_3) = \begin{bmatrix} D_2 & D_1 & D_8 \\ D_4 & D_3 & D_9 \\ 1 & 1 & C_{\beta_3} \end{bmatrix},$

where $D_8 = D_7 C_\vartheta + D_6 C_{\beta_3}$,
 $D_9 = D_7 S_\vartheta + D_6 S_{\beta_3}.$

• for the unicycle mounted mobile manipulator:

$$\dot{\xi} = \bar{J}(q_a, \vartheta, \beta_s)\eta$$

with: $\bar{J}(q_{a1}, q_{a2}, \vartheta) = \begin{bmatrix} D_2 & D_1 & C_\vartheta & D_5 \\ D_4 & D_3 & S_\vartheta & D_6 \\ 1 & 1 & 0 & 1 \end{bmatrix},$

and the indexes and degrees can be summarized by:

indexes	car-like	unicycle
m_p	3	3
n_a	2	2
n	9	7
δ_{s_p}	1	0
δ_{m_p}	1	2
δ_{M_p}	2	2
n_p	7	5
δ_m	3	4
δ_M	4	4
D	m=3	m=3
R	2	2
$\bar{D}$	3	3
$\bar{R}$	0	1

5. CONCLUSION

In this article, the generic modelling of wheeled mobile manipulators and the study of related notions allowed us to enlight the analogies between classical manipulation and nonholonomic mobile manipulation. The classical tools from kinematics are reviewed and it is shown that wheeled mobile manipulation gives rise to new cases. The role of steering wheels is emphasized and redundancy is especially discussed.

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